NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: JULY 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
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INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1

Write true if each of the following statements is correct and write false if it is incorrect. Justify your answer.

1.1. If λ is an eigenvalue of matrix A, then $A - \lambda I$ is invertible. [3]

1.2. An $n \times n$ matrix with fewer than n linearly independent eigenvectors is not diagonalizable.

[2]

1.3. The characteristic polynomial and the minimal polynomial of a square matrix can have different irreducible factors. [2]

QUESTION 2

Show that v is an eigenvector of A and find the corresponding eigenvalue.

$A = \begin{bmatrix} 4 & -2 \\ 5 & -7 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$. [5]

QUESTION 3

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2z \\ x + z \end{bmatrix}$.

3.1. Show that T is linear. [13]

3.2. Find the translation matrix A of T. [7]

3.3. Use the result in (3.2) to find $T\left(\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}\right)$. [4]

QUESTION 4

Let $T_1(x_1, x_2, x_3) = (4x_1 + x_3, -2x_1 + x_2, -x_1 - 3x_2)$ and $T_2(x_1, x_2, x_3) = (x_1 + 2x_2, -x_3, 4x_1 - x_3)$.

4.1. Find the standard matrices of T_1 and T_2. [12]

4.2. Use the result in (4.1) to find the standard matrices of $T_2 \circ T_1$. [5]

QUESTION 5

Let T be a linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (3x-z, 3y + 2z, x + y + z)$ and

$\mathcal{B} = \{v_1, v_2, v_3\}$ be a basis of $\mathbb{R}^3$ in which $v_1 = (1, 0, 1), v_2 = (0, 1, 2)$ and $v_3 = (1, 1, 0)$.

5.1. Find the coordinate vector $[v]_{\mathcal{B}}$ of v where v = (a, b, c) is any vector in $\mathbb{R}^3$. [13]

5.2. Use the result in (5.1) to find the coordinate vector of the vector v = (1, 2, −1) with respect to the basis $\mathcal{B}$. [4]

QUESTION 6

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

6.1. Find the eigenvalues of A. [11]

6.2. Find the eigenspace corresponding to the largest eigenvalue in (6.1). [9]

QUESTION 7

Find the quadratic form q(X) that corresponds to the symmetric matrix

$A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & -1 & 2 \\ -2 & 2 & 0 \end{bmatrix}$. [10]

END OF SUPPLEMENTARY/ SECOND OPPORTUNITY EXAMINATION QUESTION PAPER